Monte Carlo Simulation for Option Pricing

G. Barco, D. Pinheiro

Department of Mathematics, CUNY Brooklyn College
Tow Mentoring and Research Program

Problem we are trying to solve:

The pricing of various contracts made upon real life assets that give us an *option* buy the asset at a future time

Contract Examples: European Call and Put Options, Barriers, Asians

Assets: Stocks, Commodities, Indexes

All can be modeled by a process called a Geometric Brownian Motion (GBM)

Why do we care about pricing contracts?

Utilizing these contracts allow us to grow and maintain our wealth

We desire to pay the lowest cost for entry, gain high payoff, and minimize possible losses

The origins of the *contracts* stems from ‘futures’ which are more rigid; making you **committed** to a trade of goods. Options are more flexible

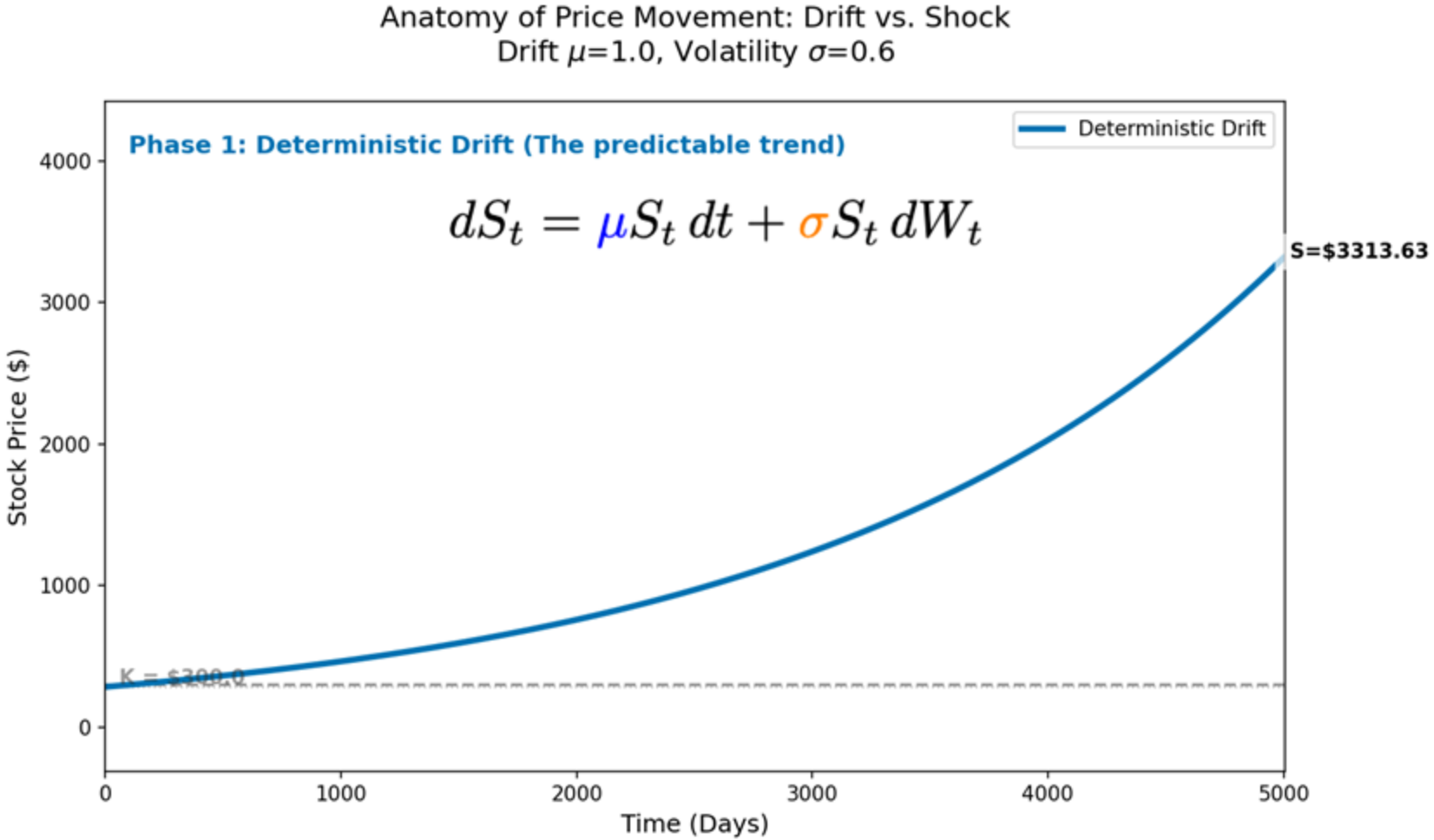
Methodology: Mathematical Model

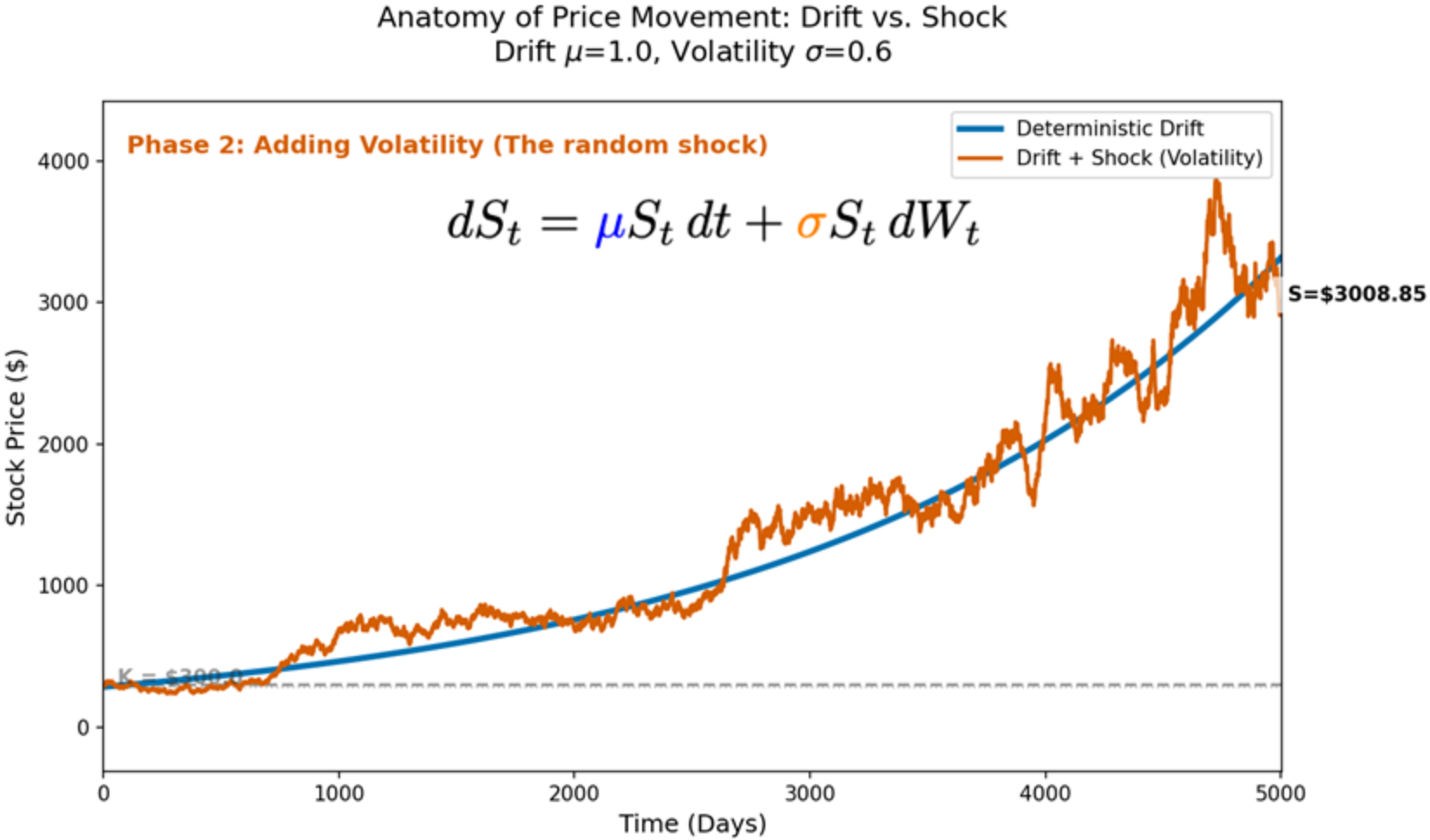
Stochastic Differential Equation (Geometric Brownian Motion)

$$dS_t = \mu S_t\,dt + \sigma S_t\,dW_t$$

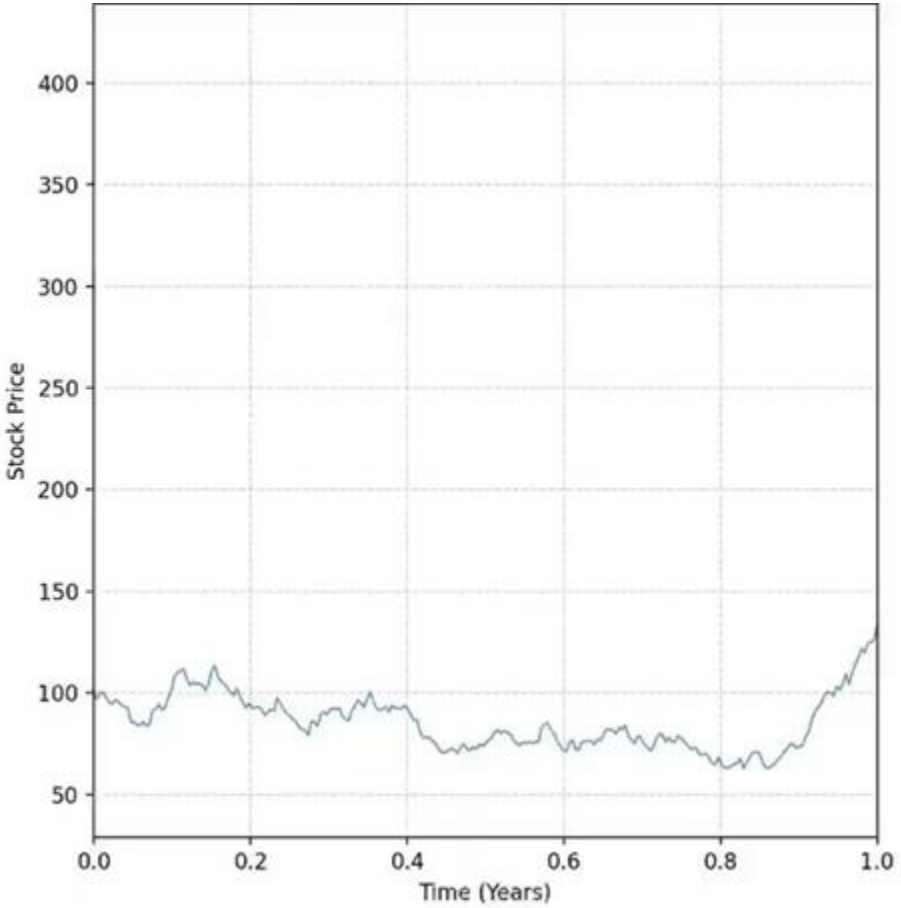
μ and **σ** measure the **average growth of price evolution** and its **uncertainty**

Important: these are constant numbers

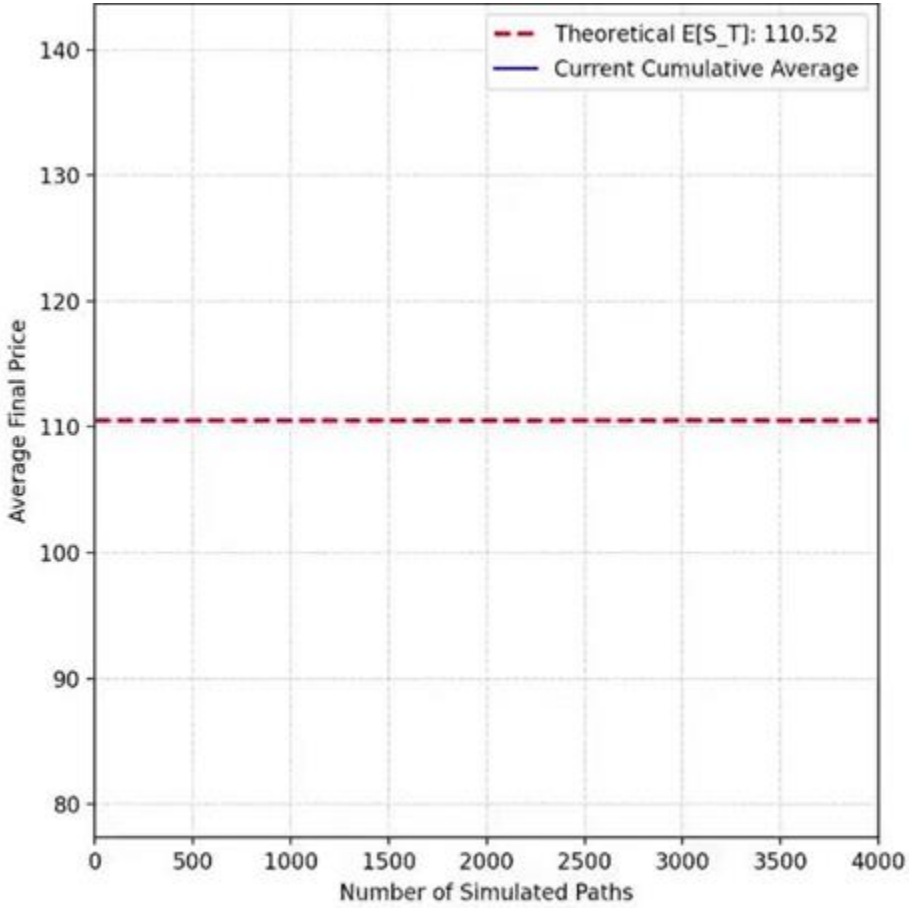

Anatomy of Price Movement: Drift vs. Shock
Drift μ=1.0, Volatility σ=0.6
Phase 1: Deterministic Drift (The predictable trend)
$dS_t = \mu S_t\,dt + \sigma S_t\,dW_t$
Deterministic Drift
S=$3313.63
Stock Price ($)
Time (Days)
0
1000
2000
3000
4000
5000


Anatomy of Price Movement: Drift vs. Shock
Drift μ=1.0, Volatility σ=0.6
Phase 2: Adding Volatility (The random shock)
$dS_t = \mu S_t\,dt + \sigma S_t\,dW_t$
Deterministic Drift
Drift + Shock (Volatility)
S=$3008.85
K = $300.0
Stock Price ($)
Time (Days)
0
1000
2000
3000
4000
5000

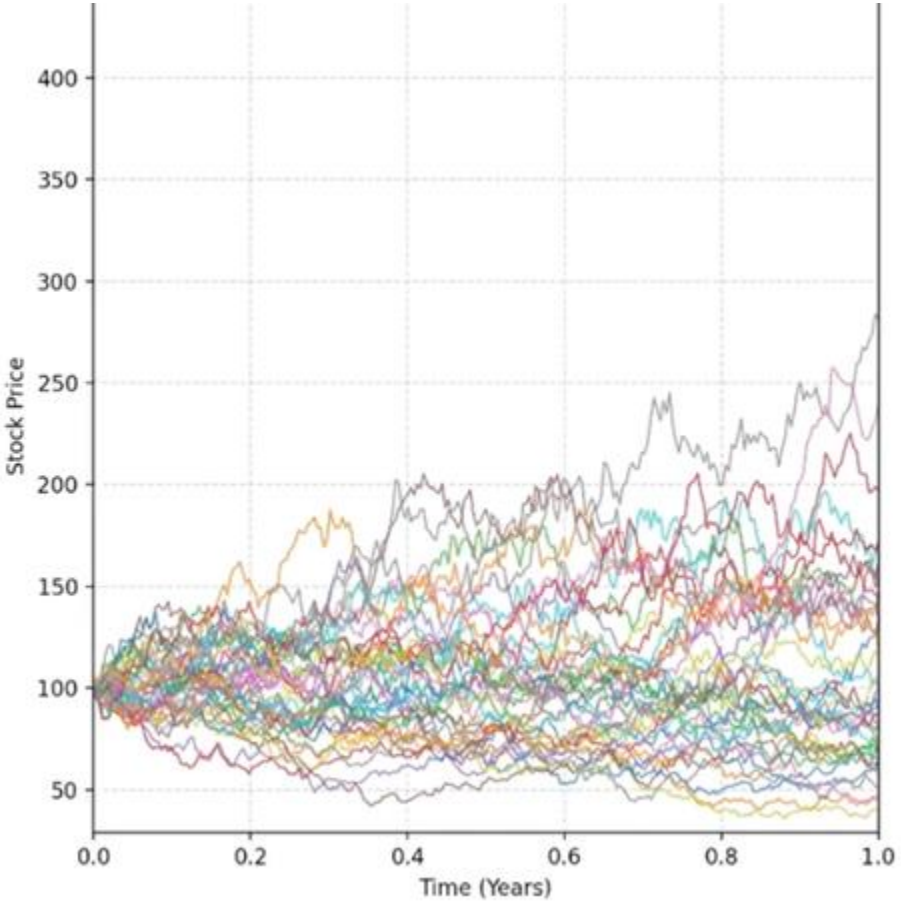
Monte Carlo Simulation



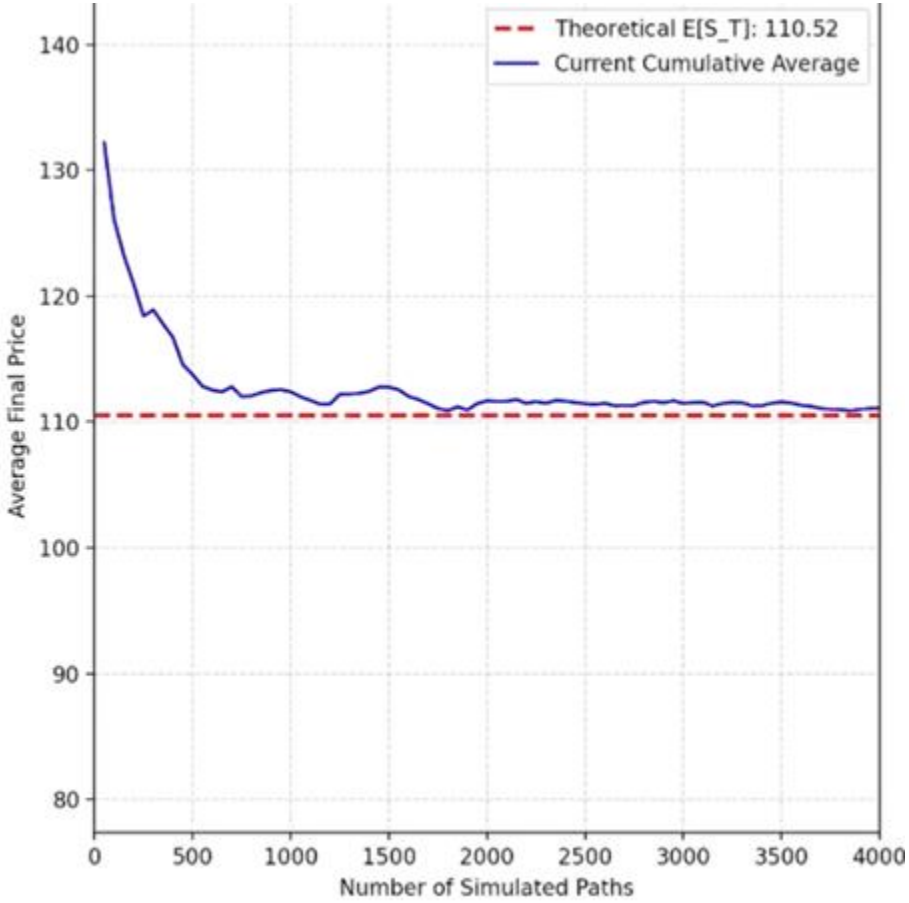
Law Of Large Numbers



Monte Carlo Simulation



Law Of Large Numbers

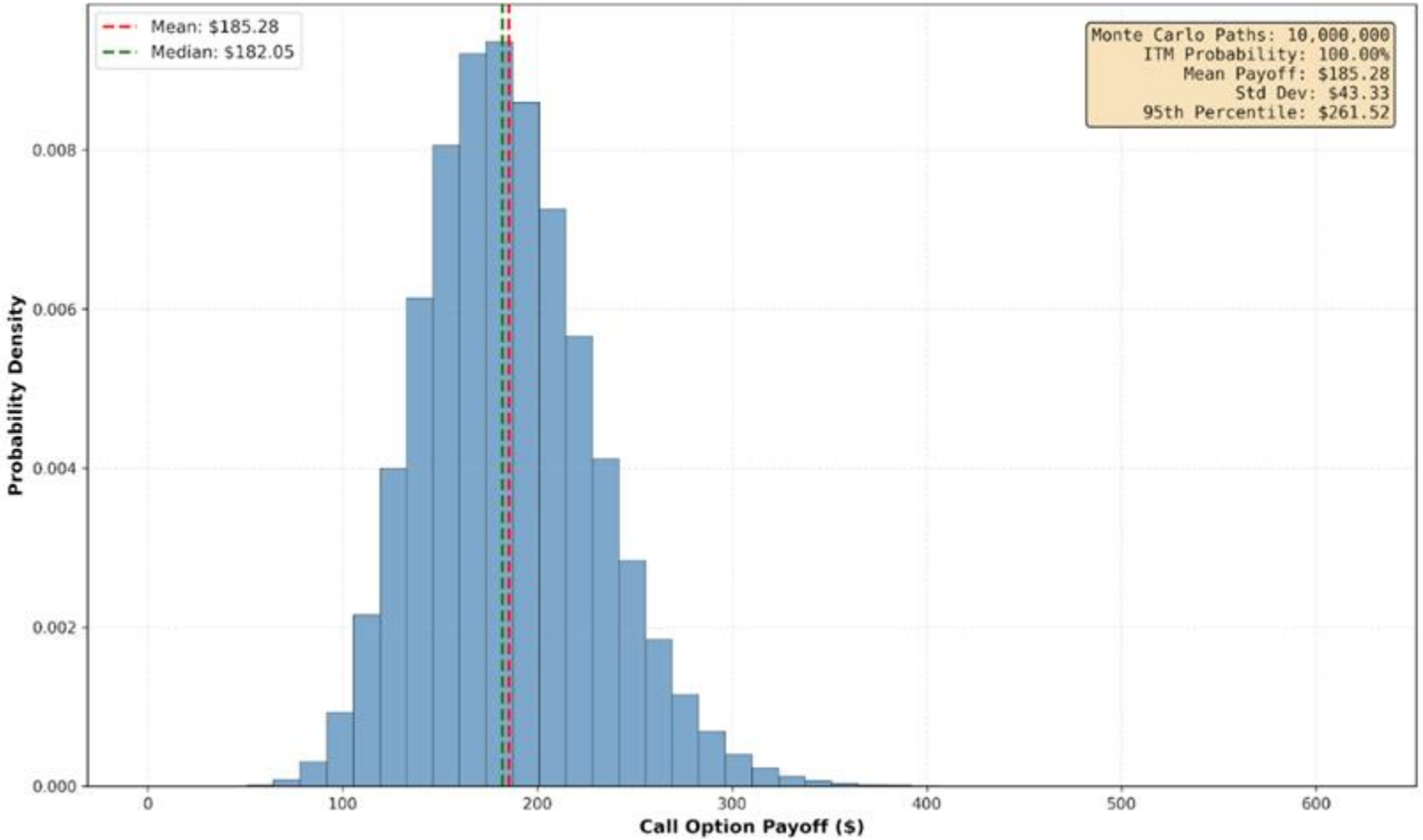


Methodology: Contract Pricing

1) **Simulate Millions of paths**
2) **Discount the path payoffs**
3) **Averaging the current values of the simulated payoffs**

$$\hat{V}_0 = \underbrace{\frac{1}{N}\sum_{i=1}^{N}}_{\text{Monte Carlo Average}} \cdot \underbrace{e^{-rT}}_{\text{Discount Factor}} \cdot \underbrace{f(S_i(T))}_{\text{Simulated Payoff}}$$

AAPL, $100 Final Price, Feb 20, 2026, Current Price = $280, E.Call Cost ≈ $180



Key Output from our research:

1) **A Program + Code**
   a) Perform statistics on market data to fit the model
2) **Simulation results:**
   a) Most Simulated Pricings converge to real prices
3) **The necessity of our research:**
   Since *most* is not good enough, we need to extend the model to study when **market uncertainty (σ)** changes

Key Output from our research: 1

Program + Code

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OPTION CHAIN PRICING (MULTIPLE STRIKES)
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Current spot price S0 ($) (https://finance.yahoo.com) [     283.100000]:
Risk-free rate r (https://www.newyorkfed.org/markets/reference-rates/effr) [       0.038900]:
Dividend yield q (https://finance.yahoo.com) [       0.003700]:
Time to maturity T (years) [       0.219178]:
Number of Monte Carlo paths [  10000000]:
Steps per year [       252]:
Random seed (0=random, Otherwise set seed) [         0]:
Absolute Minimum Strike Price [       0.000000]: 100
Absolute Maximum Strike Price [       0.000000]: 450
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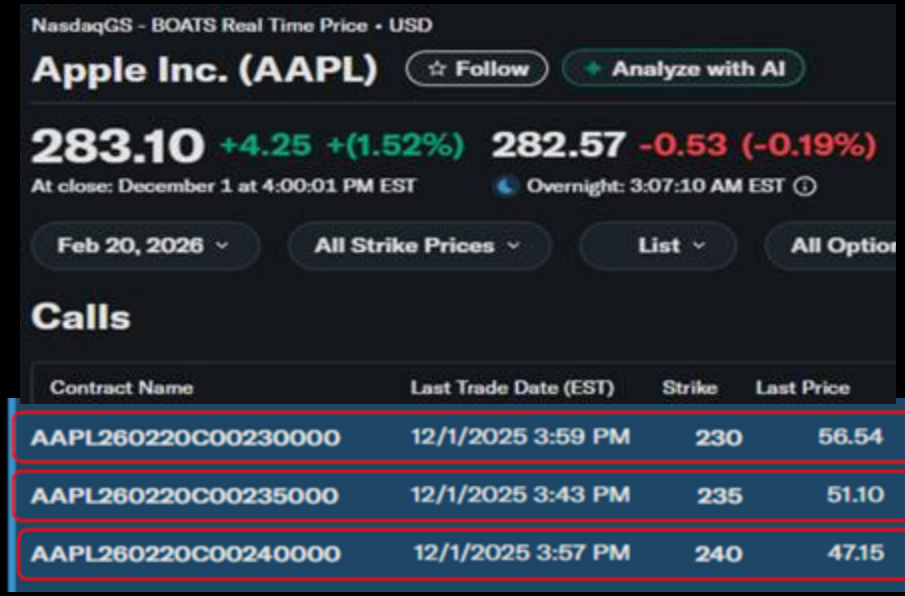
Key Output from our research: 2

Simulation Results vs Real Life

Apple Stock
Current Price: 283.10
Feb 20, 2026
European Call Option

K=	
230	56.1374
235	51.6784
240	47.3538



Key Output from our research: 3

The Necessity of Our Research

μ and **σ** measure the **average growth of price evolution** and its **uncertainty**

Our model assumed they were constant, In reality they change randomly

Further extensions: We implement more realism by allowing **σ** to vary

As **μ plays no role in real world computation**

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